Prime Factoring

When a number written as the product of prime factors, it is called the prime factorization of a number.

$$48 = 3 * 2 * 2 * 2 * 2 = 3 * 2^4$$

To make finding the prime factors easy, you need to be a master of "**The Factor Facts**" and **Divisibility Rules** for 2, 3, 5, 7 and 11. This workbook will review those skills before showing you the strategies for finding prime factors.

There are 2 attack strategies for quickly and easily finding the prime factors of a given number.

- 1. Easy Primes
- 2. Ladder

Factor Facts

There are 32 numbers that are supposed to be quick and easy to recognize the factors of. Math6.org calls these numbers, "Factor Facts" and offers matching exercises and drills to help you learn them quickly and easily.

12	18	25	32	42	50	63	80
14	20	27	35	45	54	64	81
15	21	28	36	48	56	70	90
16	24	30	40	49	60	72	100

Easy Factors

Often called the tree method, easy primes involves learning to think of an easy factor and write it as the product of its prime factors. Students will think 6 and write 2*3. So when they are thinking 6*8, they write 3*2 *2*2*2. This process isn't difficult, but it does take practice to make it so easy that it's second nature.

$$4 = 2 * 2$$

$$6 = 3 * 2$$

$$8 = 2 * 2 * 2$$

$$9 = 3 * 3$$

$$10 = 5 * 2$$

Take a look at the "Factor Facts" and notice that **all of them** are made out factors that can be easily primed.

28 is
$$7 * 4 = 7 * 2 * 2$$
; 56 is $7 * 8 = 7 * 2 * 2 * 2$; 90 is $9 * 10 = 3 * 3 * 5 * 2$

Ladders

When you don't know the "easy factors" of a number, you use the divisibility rules for 2, 3, 5, 7, and 11 to begin breaking a number down. Using the <u>divisibility rules</u> and <u>short division</u>, you can quickly find all of the prime factors of any number.

$$\begin{array}{c|c}
 & 17 \\
3 & 51 \\
2 & 102 \\
2 & 204 \\
\end{array}$$

$$204 = 17 * 3 * 2 * 2$$

The "Factor Facts"

Lesson Box

The following numbers are numbers that you should be able to look at and instantly **know** the factors of.

12	25	42	63
14	27	45	64
15	28	48	70
16	30	49	72
18	32	50	80
20	35	54	81
21	36	56	90
24	40	60	100

If **any** of these multiplication facts are still giving you pause and stumbles, please make flash cards, study "Factor Facts" @ Math6.org, or copy them 10 times each until you are able to instantly declare the factors for these numbers.

	Models	
<u>4 • 3</u> 12		9 • 6 54
<u>7 • 2</u> 14	<u>6 • 5</u> 30	<u>8 • 7</u> 56
<u>5 • 3</u> 15	<u>8 • 4</u> 32	<u>10 • 6</u> 60
<u>4 • 4</u> 16	<u>7•5</u> 35	<u>9•7</u> 63
<u>9•2</u> 18	<u>6•6</u> 36	<u>8 • 8</u> 64
<u>5 • 4</u> 20	<u>8 • 5</u> 40	<u>10 • 7</u> 70
<u>7•3</u> 21	<u>7•6</u> 42	<u>9 • 8</u> 72
<u>6 • 4</u> 24	<u>9•5</u> 45	<u>10 • 8</u> 80
<u>5 • 5</u> 25	<u>8 • 6</u> 48	9•9 81
<u>9•3</u> 27	<u>9•5</u> 49	<u>10 • 9</u> 90
<u>7 • 4</u> 28	<u>10 • 5</u> 50	<u>10 • 10</u> 100

I want you to be **much faster** and make this skill **even easier**. I don't think you need to use the multiplication sign between the factors the way that I did. As you complete the drill on this page and the drills on the next, you may drop the multiplication sign **and** remember that multiplication is commutative – **order** doesn't matter! $120 = 12 \cdot 10$; you may write $12\ 10$ on your paper and leave the multiplication sign out.

 . 42	 40	 18	 70
 60	 24	 28	 35
 . 54	 64	 100	 42
 56	 20	 63	 64
 . 30	 25	 50	 48
 . 21	 45	 . 12	 . 15
 70	 35	 16	 60
 . 27	 80	 . 72	 72
 . 49	 81	 . 14	 56
 . 48	 90	 36	 28
 . 32	 15	 90	 81

[use **Key 1** to check your answers]

Factor Facts Drills

[Key 3]	[Key 4]	[Key 5]	[Key 6]
70	64	90	14
72	70	60	32
48	72	50	25
45	12	54	56
25	40	49	90
14	15	20	21
54	45	12	45
15	48	80	16
27	42	16	20
60	24	63	28
81	56	70	24
100	60	40	60
64	32	35	54
40	18	15	72
24	30	72	48
56	49	27	70
16	25	28	35
32	27	36	18
20	80	32	40
36	90	14	36
63	14	45	64
42	28	48	42
12	16	24	15
18	50	18	63
49	20	64	81

The "Easy Primes"

Lesson Box

We need to learn to think of a few numbers as the product of their primes. The easy primes are 4, 6, 8, 9, and 10!

$$4 = 2 \cdot 2$$

$$6 = 3 \cdot 2$$

$$8 = 2 \cdot 2 \cdot 2$$

$$9 = 3 \cdot 3$$

$$10 = 5 \cdot 2$$

When you get good at thinking "10" and writing 5 • 2, the rest of the prime factoring skill will be much easier. Use the drills on this page and/or practice "Easy Primes" @ Math6.org!

Models

$$\begin{array}{ccc} 2 & = & 2 \\ \hline 3 & = & 3 \end{array}$$

$$2 \cdot 2 \cdot 2 = 8$$

$$3 \cdot 3 = 9$$

$$5 \cdot 2 = 10$$

I want you to be **much faster** and make this skill **even easier**. I don't think you need to use the multiplication sign between the factors the way that I did. As you complete the drill on this page, you may drop the multiplication sign **and** remember that multiplication is commutative – **order** doesn't matter! $6 = 3 \cdot 2$; you may write 3 2 on your paper and leave the multiplication sign out.

[use **Key 7** to check your answers]

The "Easy Factors"

Lesson Box

Now that we know the "<u>Easy Primes</u>", we put it together with the "<u>Factor Facts</u>" and easily factor these numbers! When you see a "Factor Fact" you will know the prime factorization!

$$40 = 8 \cdot 5 = 2 \cdot 2 \cdot 2 \cdot 5$$

 $42 = 7 \cdot 6 = 7 \cdot 3 \cdot 2$
 $63 = 9 \cdot 7 = 3 \cdot 3 \cdot 7$
 $72 = 9 \cdot 8 = 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$

Think of the factors and write those factors as easy primes. You will quickly and easily be able to factor most of the fractions that you will soon face! Use the drills on this page and/or practice "Easy Factors" @ Math6.org!

Models

I want you to be **much faster** and make this skill **even easier**. I don't think you need to use the multiplication sign between the factors the way that I did. As you complete the drills on this page and the next, you may drop the multiplication sign **and** remember that multiplication is commutative – **order** doesn't matter! $6 = 3 \cdot 2$; you may write 3 2 on your paper and leave the multiplication sign out.

= 30	= 36	= 18
= 56	= 21	= 35
= 48	= 25	= 40
= 24	= 63	= 42
= 16	= 81	= 54
= 45	= 70	= 80
= 15	= 32	= 12
= 14	= 50	= 64
= 60	= 90	= 100
= 72	= 20	= 49
= 28	= 27	= 24
	[use Key 8 to check your answers]	

Easy Factor Drills

[Key 9]		[Key 10]		[Key 11]	
	= 15		= 40	· 	_ = 100
	= 35		= 80		_ = 25
	= 54		= 24		= 35
	= 16		= 14		= 36
	= 36		= 81		= 81
	= 49		= 12		= 80
	= 12		= 30		= 21
	= 24		= 28		= 56
	= 81		= 42		= 70
	= 32		= 60		_ = 64
	= 72		= 27		= 24
	= 45		= 64		= 27
	= 100		= 100		_ = 30
	= 27		= 25		_ = 50
	= 20		= 15		_= 16
	= 60		= 21		_= 60
	= 42		= 90		_= 28
	= 30		= 50		_ = 49
	= 28		= 16		_ = 14
	= 70		= 20		_ = 90
	= 14		= 70		_ = 15
	= 50		= 72		_ = 45
	= 48		= 45		_ = 20
	= 56		= 36		_ = 12
	= 21		= 48		= 42

Factor Ladders

Lesson Box

When a the prime factorization of a number is not easily known using "<u>Factor Facts</u>", you apply the <u>divisibility rules</u> for 2, 3, 5, 7, and 11 to find a divisor. Use <u>short division</u> to build a "<u>Factor</u> Ladder".

- 1. Check for divisibility by 2, 3, 5, 7, and/or 11.
- 2. Use short division to find the quotient.
- 3. Repeat steps 1 and 2 on the quotient.

Continue until the quotient is a prime number.

Models

$$\begin{array}{c|c}
1 & 1 & 3 \\
2 & 2 & \underline{6}
\end{array}$$

$$226 = 113 \cdot 2$$

$$159 = 53 \cdot 3$$

$$\begin{array}{c|c}
23 \\
2 & \underline{46} \\
5 & \underline{230}
\end{array}$$

$$\begin{array}{c|c}
23 \\
7 & 161 \\
2 & 32\underline{2}
\end{array}$$

$$230 = 23 \bullet 5 \bullet 2$$

$$322 = 23 \cdot 7 \cdot 2$$

Use factor ladders to find the prime factorization of each of these numbers. Write the prime factorization on the line and check your work. Remember that multiplication is commutative so the **order** of your answers will not matter.

____ = 102

[use **Key 12** to check your answers]

Find the Prime Factorization of each of the numbers below. You should use exponents to make writing and checking your work easier.

Drill A		Drill B		Drill C
	_ = 24		= 17	= 8
	= 25		= 18	= 12
	_ = 32		= 20	= 16
	_ = 40		= 21	= 22
	= 42		= 44	= 30
	_ = 51		= 49	= 34
	= 78		= 50	= 45
	= 80		= 58	= 48
	= 81		= 63	= 64
	= 82		= 72	= 66
	= 84		= 87	= 76
	_ = 90		= 95	= 88
	_ = 91		= 98	= 100
	_ = 94		= 220	= 102
	_ = 210		= 225	= 350
	_ = 500		= 400	= 474

Several of these problems have **more than one** possible answer. Make sure you want your answer is actually wrong before you mark it wrong. Also, remember that multiplication is commutative so the order you chose doesn't matter!

[Key 1]

7 • 6	8 • 5 or 10 • 4	9 • 2 or 6 • 3	10 • 7
10 • 6	8 • 3 or 6 • 4	7 • 4	7 • 5
6 • 6 or 9 • 4	8 • 8	10 • 10	7 • 6
8•7	5 • 4 or 10 • 2	9•3	8 • 8
6 • 5	5 • 5	10 • 5	8 • 6
7 • 3	9•5	4 • 3 or 6 • 2	5•3
10 • 7	7 • 5	4 • 4 or 8 • 2	10 • 6
9•3	10 • 8	9 • 8	9 • 8
7 • 7	9•9	7 • 2	8 • 7
8•6	10 • 9	6 • 6 or 9 • 4	7 • 4
8 • 4	5•3	10 • 9	9•9

Remember that multiplication is commutative – **order** doesn't matter!

[Key 7]

5	5•2	2	3
2 • 2 • 2	2 • 2	2•2	3 • 3
5•2	3	3 • 3	2•2
2•2	2	3	5
3 • 3	2 • 2 • 2	7	3 • 2
2	3 • 2	3 • 2	2 • 2 • 2
3	5	5	2•2
7	5 • 2	2	2
3•2	5	2 • 2 • 2	7
7	2 • 2 • 2	5•2	5 • 2
3•3	3 • 2	7	3 • 3

Factor Facts Drills

[Key 3]	[Key 4]	[Key 5]	[Key 6]
7•7	8 • 8	10 • 9	7 • 2
9 • 8	10 • 7	10 • 6	8 • 4
8 • 6	9 • 8	10 • 5	5 • 5
9 • 5	4 • 3 or 6 • 2	9 • 4	8 • 7
5 • 5	8 • 5 or 10 • 4	7 • 7	10 • 9
7 • 2	5 • 3	5 • 4 or 10 • 2	7•3
8 • 7	9•5	4 • 3 or 6 • 2	9•5
5•3	8 • 6	10 • 8	6 • 3 or 8 • 2
9•3	7 • 6	4 • 4 or 8 • 2	5 • 4 or 10 • 2
10 • 6	6 • 4 or 8 • 3	9•7	7 • 4
9•9	8•7	10 • 7	6 • 4 or 8 • 3
10 • 10	10 • 6	8 • 5 or 10 • 4	10 • 6
8 • 8	8 • 4	7 • 5	9•6
8 • 5 or 10 • 4	6 • 3 or 9 • 2	5 • 3	9•8
6 • 4 or 8 • 3	6 • 5 or 10 • 3	9 • 8	8 • 6
8 • 7	7 • 7	9•3	10 • 7
4 • 4 or 8 • 2	5 • 5	7 • 4	7•5
8 • 4	9•3	6 • 6 or 9 • 4	6 • 3 or 9 • 2
5 • 4 or 10 • 2	10 • 8	8 • 4	8 • 5 or 10 • 4
6 • 6 or 9 • 4	10 • 9	7 • 2	6 • 6 or 9 • 4
9•7	7 • 2	9•5	8 • 8
7 • 6	7 • 4	8 • 6	7•6
4 • 3 or 6 • 2	6 • 3 or 8 • 2	6 • 4 or 8 • 3	5•3
6 • 3 or 8 • 2	10 • 5	6 • 3 or 9 • 2	9•7
7•7	5 • 4 or 10 • 2	8 • 8	9•9

Remember that multiplication is commutative – order doesn't matter!				
	[Key 8]			
3•2•5	3 • 2 • 3 • 2	3 • 3 • 2		
2 • 2 • 2 • 7	7 • 3	7 • 5		
2 • 2 • 2 • 3 • 2	5 • 5	2 • 2 • 2 • 5		
3 • 2 • 2 • 2	3 • 3 • 7	7 • 3 • 2		
2 • 2 • 2 • 2	3 • 3 • 3 • 3	3 • 3 • 3 • 2		
3 • 3 • 5	5 • 2 • 7	5 • 2 • 2 • 2 • 2		
5•3	2 • 2 • 2 • 2 • 2	2 • 2 • 3		
7 • 2	5 • 2 • 5	2 • 2 • 2 • 2 • 2 • 2		
5 • 2 • 3 • 2	5 • 2 • 3 • 3	5 • 2 • 5 • 2		
3 • 3 • 2 • 2 • 2	5 • 2 • 2	7•7		
7 • 2 • 2	3 • 3 • 3	3 • 2 • 2 • 2		
_		rs. Write the prime factorization on the line he order of your answers will not matter.		
43 • 3	37 • 2	127 • 1		
13 • 11	19•7	7 • 3 • 2 • 2		
7 • 7 • 3	3 • 2 • 2 • 2 • 2 • 2	59 • 2		
43 • 2	19 • 3 • 3	83 • 3		
29 • 2	17 • 3	61 • 2		
41 • 2 • 2	13 • 3 • 2 • 2	7 • 2 • 2 • 2		
17 • 3 • 2	7 • 3 • 3 • 2	17 • 2		
23 • 3	19 • 2 • 2 • 2	11 • 3 • 2 • 2		
3 • 3 • 2 • 2 • 2 • 2	5 • 3 • 3 • 2 • 2	19•2		
37 • 2 • 2	53 • 3	7 • 3 • 2 • 2 • 2		

59 • 3

31 • 3

[Key 12]

43 • 2 • 2

23 • 2

13 • 2

17 • 5 • 2

Easy Factor Drills

[Key 9]	[Key 10]	[Key 11]
5 • 3	2 • 2 • 2 • 5	5 • 2 • 5 • 2
7 • 5	5 • 2 • 2 • 2 • 2	5 • 5
3 • 3 • 3 • 2	3 • 2 • 2 • 2	7 • 5
2 • 2 • 2 • 2	7•2	3 • 2 • 3 • 2
3 • 2 • 3 • 2	3 • 3 • 3 • 3	3 • 3 • 3 • 3
7•7	2 • 2 • 3	5 • 2 • 2 • 2 • 2
2 • 2 • 3	3 • 2 • 5	7•3
3 • 2 • 2 • 2	7 • 2 • 2	2 • 2 • 2 • 7
3 • 3 • 3 • 3	7 • 3 • 2	5 • 2 • 7
2 • 2 • 2 • 2 • 2	5 • 2 • 3 • 2	2 • 2 • 2 • 2 • 2 • 2
3 • 3 • 2 • 2 • 2	3 • 3 • 3	3 • 2 • 2 • 2
3 • 3 • 5	2 • 2 • 2 • 2 • 2 • 2	3 • 3 • 3
5 • 2 • 5 • 2	5 • 2 • 5 • 2	3 • 2 • 5
3 • 3 • 3	5•5	5 • 2 • 5
5 • 2 • 2	5•3	2 • 2 • 2 • 2
5 • 2 • 3 • 2	7•3	5 • 2 • 3 • 2
7 • 3 • 2	5 • 2 • 3 • 3	7 • 2 • 2
3 • 2 • 5	5 • 2 • 5	7•7
7 • 2 • 2	2 • 2 • 2 • 2	7•2
5 • 2 • 7	5 • 2 • 2	5 • 2 • 3 • 3
7•2	5 • 2 • 7	5•3
5 • 2 • 5	3 • 3 • 2 • 2 • 2	3 • 3 • 5
2 • 2 • 2 • 3 • 2	3 • 3 • 5	5 • 2 • 2
2 • 2 • 2 • 7	3 • 2 • 3 • 2	2 • 2 • 3
7•3	2 • 2 • 2 • 3 • 2	7 • 3 • 2

Find the Prime Factorization of each of the numbers below. You should use exponents to make writing and checking your work easier.

Drill A	Drill B	Drill C
$3 \cdot 2^3$	17 • 1	2^3
5^2	$3^2 \cdot 2$	$3 \cdot 2^2$
2^5	$5 \cdot 2^2$	24
$5 \cdot 2^3$	7 • 3	11 • 2
7 • 3 • 2	$11 \cdot 2^2$	5 • 3 • 2
17 • 3	7^2	17 • 2
13 • 3 • 2	52 • 2	$5 \cdot 3^3$
5 • 2 ⁴	29 • 2	3 • 2 ⁴
3^4	$7 \cdot 3^2$	2^6
41 • 2	$3^2 \cdot 2^3$	11 • 3 • 2
$7 \cdot 3 \cdot 2^2$	29 • 3	$19 \bullet 2^2$
$5 \cdot 3^2 \cdot 2$	19 • 5	$11 \cdot 2^3$
91 • 1	$7^2 \cdot 2$	$5^2 \cdot 2^2$
47 • 2	$11 \cdot 5 \cdot 2^2$	17 • 3 • 2
7 • 5 • 3 • 2	$5^2 \cdot 3^2$	$7 \cdot 5^2 \cdot 2$
$5^3 \cdot 2^2$	$5^2 \cdot 2^4$	79 • 3 • 2